Introduction to Supersymmetry

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Lecture 5

- Grand Unification

Grand Unification

Standard Model: remnant of a larger gauge symmetry

- semi-simple group $G \to SU(3) \times SU(2) \times U(1)_Y$ at a scale $M_{\rm GUT}$
- $U(1)_Y$: non-abelian generator $\Rightarrow$ charge quantization

Main consequences:

- G has a single coupling $g \Rightarrow$ gauge coupling unification
- $\mathrm{Tr}\, Y = 0$ in every representation of $G \Rightarrow$

  quarks and leptons are generally mixed $\Rightarrow$

  B, L violation $\to$ proton decay $\Rightarrow$ $M_{\rm GUT} \gtrsim 10^{15}$ GeV

gauge coupling unification

- non abelian couplings: $g_2 = g_3$ at $M_{\rm GUT}$ but what about g_Y?
- $U(1)_Y$ should also be normalized as the non-abelian generators

representation R: ${\rm Tr}_R t_a t_b = T(R)\delta_{ab}$

compute ${\rm Tr}\, T_3^2$ and ${\rm Tr}\, Y^2$ for a complete fermion family:

	$q_{1/6}$	$u^c_{-2/3}$	$d^c_{1/3}$	$\ell_{-1/2}$	e^c_1	total
${\rm Tr}\, T_3^2$	$3\times 2\times\frac{1}{4}=\frac{3}{2}$	0	0	$2\times\frac{1}{4}=\frac{1}{2}$	0	2
${\rm Tr}\, Y^2$	$6\times\frac{1}{36}=\frac{1}{6}$	$3\times\frac{4}{9}=\frac{4}{3}$	$3\times\frac{1}{9}=\frac{1}{3}$	$2\times\frac{1}{4}=\frac{1}{2}$	1	$\frac{10}{3}$

it follows: $Y=\sqrt{\frac{5}{3}}T_1$ (non-abelian generator) $\Rightarrow$

$g_Y=\sqrt{\frac{3}{5}}g_1$ (covariant derivative: $g_Y Y = g_1 T_1$) $g_1=g_2=g_3\Rightarrow$

prediction: $\sin^2\theta_W=\frac{g_Y^2}{g_Y^2+g_2^2}=\frac{3/5}{3/5+1}=\frac{3}{8}$ at $M_{\rm GUT}$

Renormalization Group evolution

At energies $< M_{\mathrm{GUT}}$ only light SM particles contribute in the loops

running with the SM beta-functions: $\frac{d\alpha_i}{d \ln Q} = -\frac{b_i}{2\pi}\alpha_i^2$

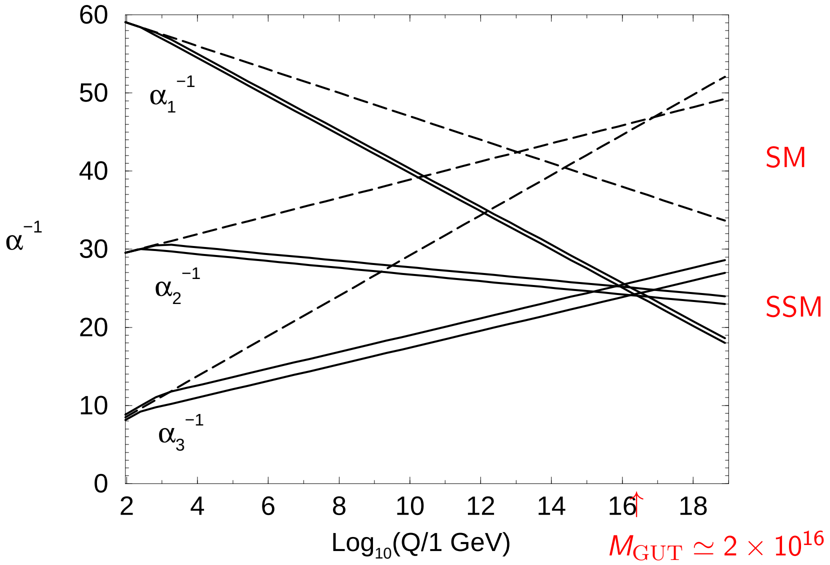
beta-function coefficients:

$$
\begin{array}{lll}
 & \text{SM} & \text{SSM} \\
b_3 = & 11 - \frac{4}{3}N_g & = 9 - 2N_g \leftarrow \text{nb of generations} \\
b_2 = & \frac{22}{3} - \frac{4}{3}N_g - \frac{1}{6}N_H & = 6 - 2N_g - \frac{1}{2}N_H \\
b_1 = & -\frac{4}{3}N_g - \frac{1}{10}N_H & = -2N_g - \frac{3}{10}N_H \leftarrow \text{nb of Higgs doublets}
\end{array}
$$

low energy data at M_Z:

$\alpha_3^{-1} = 8.50 \pm 0.14 \quad \alpha_2^{-1} = 29.57 \pm 0.02 \quad \alpha_1^{-1} = 59.00 \pm 0.02$

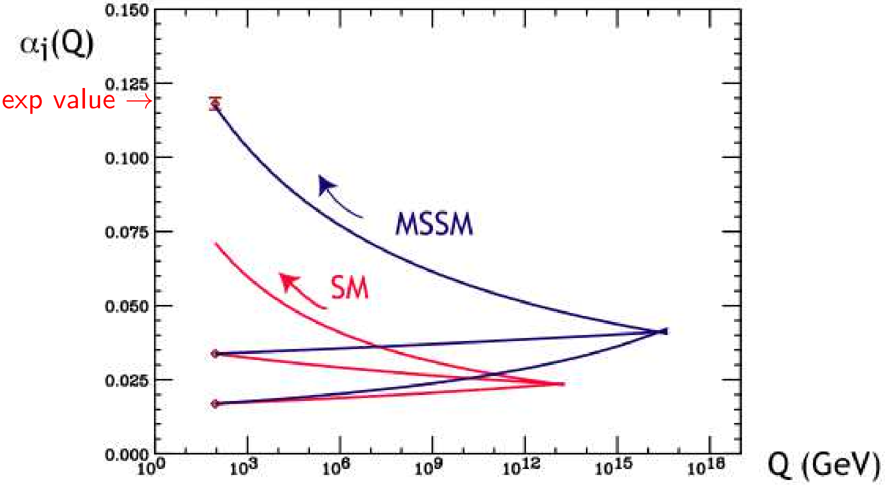
gauge coupling evolution of SM versus SSM



$M_{\rm GUT} \simeq 2 \times 10^{16}$ GeV

GUT prediction of QCD coupling

input $\alpha_{\rm em}, \sin^2\theta_W \quad \Rightarrow \quad$ output α_3



Standard Model: rank 4 $\Rightarrow$ rank$(G) \geq 4$

$\mathrm{Tr}Q$ of SM representations:

$q \to (\frac{2}{3} - \frac{1}{3}) \times 3 = 1 \quad u^c \to -2 \quad d^c \to 1 \quad \ell \to -1 \quad e^c \to 1 \quad \Rightarrow$

traceless combinations: $(u^c q d^c)(\ell e^c), (u^c d^c e^c)(q\ell), (u^c q e^c)(d^c \ell)$

only possibility: $SU(5)$ **10** $\bar{\mathbf{5}}$

SM embedding in $SU(5)$: generators 5×5 traceless matrices

$$\begin{pmatrix} SU(3) & 0 \\ 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 \\ 0 & SU(2) \end{pmatrix} \quad U(1): T_1 = c \begin{pmatrix} -\frac{1}{3}\mathbb{1}_{3\times 3} & 0 \\ 0 & \frac{1}{2}\mathbb{1}_{2\times 2} \end{pmatrix}$$

$$\mathrm{Tr}\, T_1 = 0 \quad \mathrm{Tr}\, T_1^2 = \frac{1}{2} \quad \Rightarrow \quad c = \sqrt{\frac{3}{5}} \quad T_1 = cY$$

SM embedding in $SU(5) \supset SU(3) \times SU(2) \times U(1)$

fermions: $\bar{\mathbf{5}} = (\bar{\mathbf{3}}, 1)_{1/3} + (1, \mathbf{2})_{-1/2}$ $\quad \mathbf{10} = (\mathbf{3}, \mathbf{2})_{1/6} + (\bar{\mathbf{3}}, 1)_{-2/3} + (1, 1)_1$

d^c $\quad \ell$ $\quad q$ $\quad u^c$ $\quad e^c$

$$\bar{\mathbf{5}} = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e^- \\ -\nu_e \end{pmatrix} \qquad \mathbf{10} = \begin{pmatrix} 0 & u_3^c & -u_2^c & -u_1 & -d_1 \\ -u_3^c & 0 & u_1^c & -u_2 & -d_2 \\ u_2^c & -u_1^c & 0 & -u_3 & -d_3 \\ u_1 & u_2 & u_3 & 0 & -e^c \\ d_1 & d_2 & d_3 & e^c & 0 \end{pmatrix}$$

adjoint $\mathbf{24} = (\mathbf{8}, 1)_0 + (1, \mathbf{3})_0 + (1, 1)_0 + (\mathbf{3}, \mathbf{2})_{-5/6} + (\bar{\mathbf{3}}, \bar{\mathbf{2}})_{5/6}$

gluons $\quad W^{\pm,3}$ $\quad B$ $\quad \begin{pmatrix} X \\ Y \end{pmatrix}$ $\quad \begin{matrix} Q = 4/3 \\ Q = 1/3 \end{matrix}$

X, Y: 12 more generators
$$\begin{pmatrix} & & & \bar{X}_1 & \bar{Y}_1 \\ & 0 & & \bar{X}_2 & \bar{Y}_2 \\ & & & \bar{X}_3 & \bar{Y}_3 \\ X_1 & X_2 & X_3 & & \\ Y_1 & Y_2 & Y_3 & & \end{pmatrix}$$

GUT symmetry breaking

Higgs in **24** $SU(5)$ adjoint: Σ

$$\langle\Sigma\rangle = V\,Y \neq 0 \quad \Rightarrow \quad SU(5) \to SU(3) \times SU(2) \times U(1)$$

- 12 Goldstone bosons eaten by $X, Y \to$ massive
- massive physical higgses: color octet + weak triplet + singlet

EW symmetry breaking: need a pair of $\mathbf{5} + \bar{\mathbf{5}}$ higgses $H, \bar{H}$

$$H = \begin{pmatrix} d_H \\ h \end{pmatrix} \begin{matrix} \leftarrow & \text{Higgs triplet with quantum numbers of } d \text{ quark} \\ \leftarrow & \text{Higgs doublet} \end{matrix}$$

$$\langle H \rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad \Rightarrow \quad SU(2) \times U(1) \to U(1)_{\rm em}$$

fermion masses

SUSY $SU(5)$: $H = \begin{pmatrix} d_H \\ H_2 \end{pmatrix}$ $\hat{H} = \begin{pmatrix} d_H^c \\ H_1 \end{pmatrix}$

Yukawa couplings: $\lambda_u \mathbf{10}\,\mathbf{10}\,\mathbf{5}_H + \lambda_d \mathbf{10}\,\bar{\mathbf{5}}\,\bar{\mathbf{5}}_H$

$\mathbf{10} = (q, u^c, e^c)$ $\bar{\mathbf{5}} = (d^c, \ell)$ $\mathbf{5}_H = (d_H, H_2)$ $\bar{\mathbf{5}}_H = (d_H^c, H_1)$ $\Rightarrow$

$\lambda_u \; : \; q u^c H_2 \quad + u^c e^c d_H + q q d_H$

$\lambda_d \; : \; q d^c H_1 + e^c \ell H_1 + u^c d^c d_H^c + q \ell d_H^c$

proton decay

$\Rightarrow \quad m_b = m_\tau$ at $M_{\rm GUT}$

RG evolution at low energies $\rightarrow$ correct prediction for m_b/m_τ

however it fails for the first two generations m_s/m_μ and m_d/m_e

gauge hierarchy

SUSY $SU(5)$: $H = \begin{pmatrix} d_H \\ H_2 \end{pmatrix} \quad \hat{H} = \begin{pmatrix} d_H^c \\ H_1 \end{pmatrix}$

general superpotential: $W = M_{\rm GUT} {\rm Tr}\, \Sigma^2 + \lambda {\rm Tr}\, \Sigma^3 + MH\hat{H} + \rho H \Sigma \hat{H}$

$\Rightarrow SU(5)$ breaking: $\langle \Sigma \rangle = V\, Y \neq 0$ makes H superheavy:

$$M(d_H d_H^c + H_2 H_1) + \rho V \left(-\tfrac{1}{3} d_H d_H^c + \tfrac{1}{2} H_2 H_1\right)$$
$$= \left(M - \tfrac{\rho}{3} V\right) d_H d_H^c + \left(M + \tfrac{\rho}{2} V\right) H_2 H_1 \quad \Rightarrow$$

fine-tuning to keep the EW Higgs doublets light:

$\left(M + \frac{\rho}{2} V\right) = \mu \sim \mathcal{O}(m_W)$ with $M, V \sim \mathcal{O}(M_{\rm GUT})$

Higgs triplet d_H, d_H^c: proton decay via dim-5 operators $\Rightarrow$ keep superheavy

$\rightarrow$ doublet/triplet splitting problem

$SO(10)$ grand unification

The only GUT group of rank 5:

- all fermions of a generation in a single representation

  $SU(5)$ decomposition: $\mathbf{16} = \mathbf{10} + \bar{\mathbf{5}} + 1 \rightarrow \nu^c$

- includes R-neutrino $\nu^c \Rrightarrow$ theory of neutrino masses
- EW Higgs: $\mathbf{10}_H = \mathbf{5}_H + \bar{\mathbf{5}}_H \rightarrow$ Yukawa couplings: $\mathbf{16}\,\mathbf{16}\,\mathbf{10}_H$
- $B - L$ is an $SO(10)$ generator
- Higgs sector becomes complicated

Advantages of SUSY

- natural elementary scalars
- gauge coupling unification: theory perturbative up to the GUT scale
- LSP: natural dark matter candidate
- extension of space-time symmetry: new Grassmann dimensions
- attractive mechanism of Electroweak Symmetry Breaking
- prediction of light Higgs
- rich spectrum of new particles within LHC reach

Problems of SUSY

- too many parameters: soft breaking terms

  SUSY breaking mechanism $\Rightarrow$ dynamical aspect of the hierarchy

  $+$ theory of soft terms

- SM global symmetries are not automatic

  B, L from R-parity, conditions on soft terms for FCNC suppression

- SUSY GUTs: no satisfactory model

  doublet/splitting, large Higgs reps, strong coupling above $M_{\rm GUT}$

- μ problem: SUSY mass parameter but of the order of the soft terms

- SUSY not yet discovered $\Rightarrow$ already fine-tuning at a %-per mille level

  'little' hierarchy problem

proposals for the μ problem

- NMSSM: extra singlet σ coupled to higgses

$$\delta W = \lambda_1 \sigma H_1 H_2 + \lambda_2 \sigma^3 : \ \langle \sigma \rangle \neq 0 \Rightarrow \mu\text{-term generation}$$

- dim-5 effective operator from high-energy physics in the Kähler potential

$$\delta K = \frac{1}{M} \int d^4\theta S^\dagger H_1 H_2 : \ \langle F_S \rangle \neq 0 \Rightarrow \mu = \frac{\langle F_S^\dagger \rangle}{M} \quad \langle S^\dagger \rangle = \langle F_S^\dagger \rangle \bar{\theta}^2$$

e.g. $M = M_{\rm Planck} \quad \langle F_S \rangle^{1/2} \simeq 10^{11}$ GeV $\Rightarrow \mu \sim \mathcal{O}(\rm TeV)$

- However (H_1, H_2) is a non chiral state:

  why is massless in a fundamental theory?

Little hierarchy problem

minimum of the potential: $m_Z^2 = 2\frac{m_1^1 - m_2^2 \tan_\beta^2}{\tan^2\beta - 1} \sim -2m_2^2 + \cdots$

RG evolution: $m_2^2 = m_2^2(M_{\rm GUT}) - \frac{3\lambda_t^2}{4\pi^2} m_{\tilde t}^2 \ln\frac{M_{\rm GUT}}{m_{\tilde t}} + \cdots$

$$\sim m_2^2(M_{\rm GUT}) - \mathcal{O}(1) m_{\tilde t}^2 + \cdots$$

On the other hand: upper bound on the Higgs mass:

$$m_h^2 \lesssim m_Z^2 \cos^2 2\beta + \frac{3}{(4\pi)^2}\frac{m_t^4}{v^2}\left[\ln\frac{m_{\tilde t}^2}{m_t^2} + \frac{A_t^2}{m_{\tilde t}^2}\left(1 - \frac{A_t^2}{12 m_{\tilde t}^2}\right)\right] \lesssim (130 GeV)^2$$

$m_h \simeq 126$ GeV $\Rightarrow m_{\tilde t} \simeq 3$ TeV or $A_t \simeq 3m_{\tilde t} \simeq 1.5$ TeV

$\Rightarrow$ % to a few ‰ fine-tuning is needed in m_Z^2

Reduce the fine-tuning

- minimize radiative corrections

  $M_{\rm GUT} \to \Lambda$: low messenger scale (gauge mediation)

  $$\delta m_{\tilde{t}}^2 = \frac{8\alpha_s}{3\pi} M_3^2 \ln \frac{\Lambda}{M_3} + \cdots$$

- increase the tree-level upper bound $\Rightarrow$ extend the MSSM

  extra fields beyond LHC reach $\to$ effective field theory approach

- Low scale SUSY breaking $\Rightarrow$ extend MSSM with the goldstino

  $\to$ Non linear MSSM

- $\cdots$

Split supersymmetry

$$\text{sparticles} \begin{cases} \text{scalars} & : & \text{heavy} & & \text{squarks and sleptons} \\ \text{fermions} & : & \text{light} & (\text{TeV}) & \text{gauginos and higgsinos} \end{cases}$$

- natural splitting: gauginos, higgsinos carry R-symmetry, scalars do not
- gauge coupling unification is preserved

  squarks + sleptons form complete $SU(5)$ multiplets $\Rightarrow$

  same contribution to all 1-loop beta-functions

  relative velocities of energy evolution unchanged
- Dark Matter candidate is kept

  neutralino combination of bino-wino-higgsino
- mass hierarchy problem comes back

  (stop - top) contribution to the Higgs mass becomes huge

Split supersymmetry: benefits

- number of low energy parameters is reduced significantly

  gaugino masses M_1, M_2, M_3 and Higgs scalar masses $m_1, m_2, B\mu$

  no soft sfermion masses, no A-terms
- global symmetries of the Standard Model appear again:

  B/L symmetry, no FCNC, etc
- distinct experimental signatures
- experimentally allowed Higgs mass $\Rightarrow$ 'moderate' split

  $m_0 \sim$ few - thousands TeV

  e.g. gauginos: a loop factor lighter than scalars ($\sim m_{3/2}$)

Split supersymmetry: signatures

- squarks superheavy $\Rightarrow$ long lived gluino

$$\tau_{gl} \simeq \left(3 \times 10^{-2}\mathrm{s}\right) \left(\frac{m_0}{10^9\mathrm{GeV}}\right)^4 \left(\frac{1\mathrm{TeV}}{M_3}\right)^5$$

$\Rightarrow$ displaced vertices   late decays captured near the detector, etc

- susy unification of 5 couplings at m_0:

$$\Delta\mathcal{L} = \sqrt{2}g_u H^\dagger \tilde{W}\psi_u + \sqrt{2}g_d H\tilde{W}\psi_d + \frac{1}{\sqrt{2}}g'_u H^\dagger \tilde{B}\psi_u - \frac{1}{\sqrt{2}}g'_d H\tilde{B}\psi_d - \frac{\lambda}{2}(H^\dagger H)^2$$

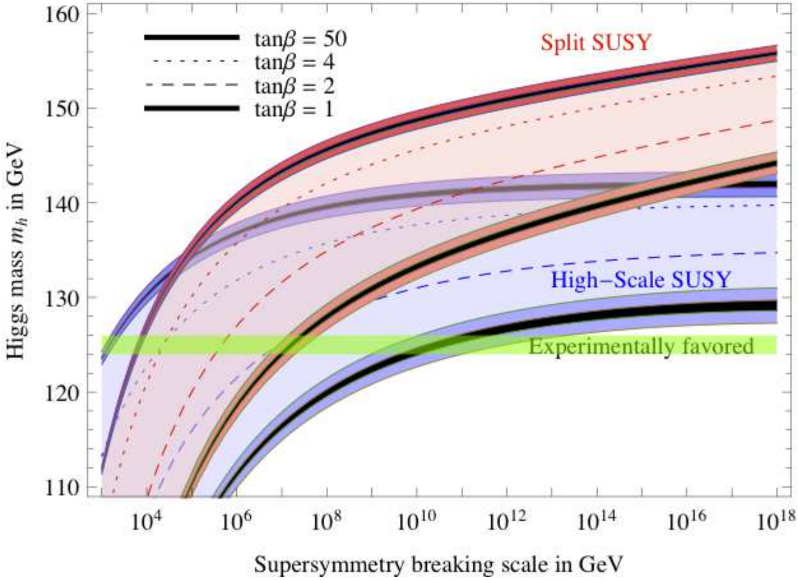
higgsinos (ψ_d, ψ_u)

susy relations: $g_u = g_2 \sin\beta$, $g_d = g_2 \cos\beta$, $g'_u = g_Y \sin\beta$

$$g'_d = g_2 \cos\beta, \; \lambda = \frac{1}{4}(g_2^2 + g_Y^2)\cos^2 2\beta$$

$\Rightarrow$ 5 relations in terms of one parameter

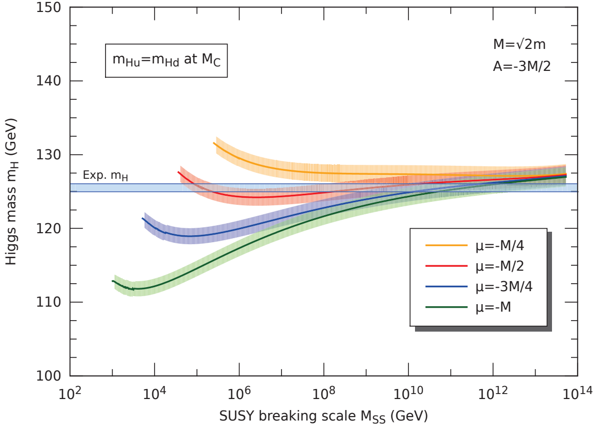
Predicted range for the Higgs mass



SUSY : $\lambda = 0 \Rightarrow \sin\beta = 1$

$H_{SM} = \sin\beta\, H_u + \cos\beta\, H_d^*$ $\qquad$ $\lambda = \frac{1}{8}(g_2^2 + g'^2)\cos^2 2\beta$

$\lambda = 0$ at a scale $\geq 10^{10}$ GeV $\Rightarrow m_H = 126 \pm 3$ GeV



e.g. for universal $\sqrt{2}m = M = M_{SS}$, $A = -3/2M$