Introduction to Supersymmetry

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Lectures 3,4

- Supersymmetric Standard Model

Supersymmetric Standard Model (SSM)

Gauge bosons

- $SU(3)$ gluons $G_\mu^{a=1,\dots,8}$ $\longrightarrow$ gluinos $\tilde{g}$
- $SU(2)$ W-bosons $W_\mu^\pm, W_\mu^3$ $\longrightarrow$ winos $\tilde{w}^\pm, \tilde{w}^3$
- $U(1)$ hypercharge B_μ $\longrightarrow$ bino $\tilde{b}$

Matter (L-handed)

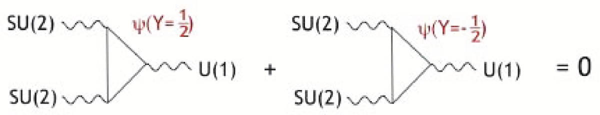
- quarks $q = \begin{pmatrix} u \\ d \end{pmatrix}_{1/6}, u^c_{-2/3}, d^c_{1/3}$ $\longrightarrow$ squarks $\tilde{q} = \begin{pmatrix} \tilde{u}_L \\ \tilde{d}_L \end{pmatrix}, \tilde{u}_R, \tilde{d}_R$
- leptons $\ell = \begin{pmatrix} \nu \\ e \end{pmatrix}_{-1/2}, e^c_1, \nu^c_0$ $\longrightarrow$ sleptons $\tilde{\ell} = \begin{pmatrix} \tilde{\nu}_L \\ \tilde{e}_L \end{pmatrix}, \tilde{e}_R, \tilde{\nu}_R$

Higgs $\longrightarrow$ higgsinos

- H_1 $Y = -1/2$ like ℓ $\longrightarrow$ $\tilde{H}_1$
- H_2 $Y = +1/2$ like $\bar{\ell}$ $\longrightarrow$ $\tilde{H}_2$

for every particle → sparticle: not present in the SM

- Can Higgs boson be spartner of a lepton ℓ ? No
  - $\langle H \rangle \neq 0$ would break lepton number (L)
  - Yukawa's: either break L or don't exist $W = \ell e^c \ell, qd^c \ell$ no $qu^c \ell$
- H_1 or H_2 ? both
  - cancel the hypercharge anomaly:



  - obtain all necessary Yukawa couplings: $qu^c H_2, qd^c H_1, \ell e^c H_1$

in SM $H_1 = H_2^\dagger$: forbidden in SUSY due to W analyticity

SSM Lagrangian

- $\mathcal{L}_{\text{gauge}} = \frac{1}{4}\int d^2\theta \, \mathrm{Tr}\mathcal{W}^2 + \mathrm{h.c.}$ $\Rightarrow$ usual gauge kinetic terms

  ↖ $SU(3) \times SU(2) \times U(1)$

- $\mathcal{L}_K = \int d^4\theta \sum_{\text{matter fields}} \Phi_q^\dagger e^{-qV} \Phi_q$ $\Rightarrow$ usual matter+higgs kinetic terms

  ↖ charges/generators of $SU(3) \times SU(2) \times U(1)$

  $\Rightarrow$ New supersymmetric gauge interactions

  - all vertices controlled by the gauge couplings
  - quartic scalar vertices from the D-terms
  - gauge "Yukawa" couplings fermion-gaugino-sfermion

- Superpotential $\int d^2\theta \, W + \mathrm{h.c.}$

$$W = (q\lambda_u u^c)H_2 + (q\lambda_d d^c)H_1 + (\ell\lambda_e e^c)H_1 + \mu H_1 H_2$$

↗ Yukawa matrices in the flavor space ↗ higgsino mass

  - new quartic scalar vertices from F-terms but not quartic Higgs potential

Higgs potential

$$\mathcal{V} = \mathcal{V}_F + \mathcal{V}_D \qquad \mathcal{V}_F = \sum_{i=1,2} \left| \frac{\partial W}{\partial H_i} \right|^2 = \mu^2 \left(|H_1|^2 + |H_2|^2 \right)$$

$$\mathcal{V}_D = \frac{1}{2} \sum_a g_a^2 \left(H_i^\dagger t^a H_i \right)^2 = \frac{g_2^2}{8} \left(H_1^\dagger \vec{\sigma} H_1 + H_2^\dagger \vec{\sigma} H_2 \right)^2 + \frac{g_Y^2}{8} \left(|H_1|^2 - |H_2|^2 \right)^2$$

$$\Rightarrow \mathcal{V}_{\text{neutral}} = \mu^2 \left(|H_1^0|^2 + |H_2^0|^2 \right) + \frac{g_2^2 + g_Y^2}{8} \left(|H_1^0|^2 - |H_2^0|^2 \right)^2$$

$$H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} \qquad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}$$

- 2 Higgs doublets $\Rightarrow$ after EW symmetry breaking: $W^\pm, Z$ massive

  1 charged scalar $H^\pm$ + 3 neutral: 2 CP-even H, h, 1 CP-odd A

- quartic coupling is predicted but 2 higgses $\quad v_2/v_1 \equiv \tan\beta$

W is not the most general renormalizable

B and L conservation is not automatic as in the SM $\rightarrow$ missing terms:

- $H_1 \leftrightarrow \ell \quad \Rightarrow \quad$ L-number violation

  $qd^c\ell \quad \ell e^c\ell \quad H_1e^cH_1 \quad \ell H_2$: $\Delta L = \pm 1$

- B-number violation: $d^cd^cu^c \quad \Delta B = 3 \times \frac{1}{3} = 1$

Discrete symmetry to forbid them: R-parity $\quad \theta$ odd

$\nwarrow$ discrete subgroup of the R-symmetry

$R = (-)^{3B+L+2J}$ = sparticle parity

gauge + Higgs superfields: even, matter: odd

Consequences of R-parity

- sparticles are produced in pairs
- Lightest Supersymmetric Particle (LSP) is stable

  typically a neutralino (mixture of bino, wino and neutral higgsinos) $\Rightarrow$

- SUSY collider signal: events with missing energy
- LSP is a natural dark matter candidate

  WIMP: weakly interacting massive particle

sparticles have not been observed $\Rightarrow$ supersymmetry must be broken

e.g. $m_{\tilde{e}} = m_e \simeq 0.5$ MeV   but $m_{\tilde{e}}|_{\rm exp} \gtrsim$ a few 100 GeV

Properties of spontaneous SUSY breaking

- Sum rule: $\mathrm{Str}\mathcal{M}^2 = \sum_{\text{bosons}} m_b^2 - \sum_{\text{fermions}} m_f^2 = 0 \qquad \sum_J (-)^{2J}(2J+1)m_J^2 = 0$

no quadratic divergence in the vacuum energy

incompatible with experimental limits

e.g. for charged leptons: $m_{\tilde{e}_L}^2 + m_{\tilde{e}_R}^2 = 2m_e^2 \quad \Rightarrow$

all slepton masses $\lesssim 2$ GeV ! similarly d-squark masses $\lesssim 5$ GeV

- Massless goldstino: $\delta\psi = -\sqrt{2}\,\langle F\rangle\,\xi + \cdots \qquad \delta\lambda = \langle D\rangle\,\xi + \cdots$

analog to Goldstone boson: $\delta\phi = c + \cdots$

it should also have derivative couplings $\Rightarrow$ cannot be known fermion

fortunately: in the presence of gravity goldstino is eaten by the gravitino

to form massive spin-3/2 $\rightarrow$ superhiggs phenomenon

Soft supersymmetry breaking

Add all possible breaking terms that preserve the good SUSY behavior

$\Rightarrow$ they should have positive mass dimensions

can be generated if SUSY is spontaneously broken in a different sector

and mediated to the SM by gauge interactions or gravity

$$m_{\rm susy} \sim \frac{\langle F \rangle}{M} \quad \text{or} \quad \frac{\langle D \rangle}{M} \quad \sim \frac{\Lambda^2}{M}$$

M: messengers mass or $M_{\rm Planck}$ Λ: SU$\!\!\!/$SY scale in the extra sector

if $M = M_{\rm Pl} \Rightarrow \Lambda \sim 10^{11}$ GeV so that $m_{\rm susy} \sim 1$ TeV

• the breaking in the extra/hidden sector can be dynamical

e.g. strongly interacting super Yang-Mills $\Rightarrow \Lambda$: gaugino condensation scale $\langle \lambda\lambda \rangle$

$\rightarrow$ dynamical explanation of the origin of SU$\!\!\!/$SY scale

Obtain the general soft terms

must have positive dimensions: masses and trilinear scalar terms $m\phi^3$

necessary but not sufficient condition $\rightarrow$ general rule:

- Introduce an auxiliary chiral superfield S with only F-component

$$S \equiv m_{\rm susy}\theta^2 \; : \; \text{spurion (dimensionless)}$$

- Promote all couplings of the supersymmetric Lagrangian to S-dependent functions/superfields

1) Matter kinetic terms: $\int d^4\theta \, \Phi^\dagger\Phi \rightarrow \int d^4\theta \, Z_\Phi(S, S^\dagger) \, \Phi^\dagger\Phi$

$Z_\Phi(S, S^\dagger) = 1 + z_\phi SS^\dagger$ up to analytic/antianalytic redefinitions

$$\Phi \rightarrow (1 + cS)\Phi, \;\; \Phi^\dagger \rightarrow (1 + c'S^\dagger)\Phi^\dagger$$

$\Rightarrow$ scalar masses: $m^2_{\rm susy} z_i |\phi_i|^2 \qquad \rightarrow m_0^2$

2) Gauge kinetic terms $\int d^2\theta\, \mathcal{W}^2 \to \int d^2\theta\, Z_{\mathcal{W}}(S)\, \mathcal{W}^2$

$Z_{\mathcal{W}}(S) = 1 + z_{\mathcal{W}} S \Rightarrow$ gaugino masses $m_{\rm susy} z_a \lambda^a \lambda^a \quad \to m_{1/2}$

3) Superpotential $\int d^2\theta\, W(\Phi) \to \int d^2\theta\, w(S) W(\Phi) \quad w(S) = 1 + \omega S$

$\Rightarrow m_{\rm susy} \omega_i W_i(\phi)$ for $W = \sum_i W_i$

$W_{\rm SSM} \to B\mu H_1 H_2 + \tilde{q}\mathbf{A_u}\tilde{u}^c H_2 + \tilde{q}\mathbf{A_d}\tilde{d}^c H_1 + \tilde{\ell}\mathbf{A_e}\tilde{e}^c H_1$

↖ matrices in flavor space

trilinear analytic scalar interactions $\quad \phi^3$ but not $\phi^2\phi^*$

$\Rightarrow$ Too many soft parameters! over 100

Exp constraints: Flavor is not automatically conserved as in SM

soft scalar masses and A-terms $\to$ important FCNC

Reducing the parameter space

Simple phenomenological conditions to suppress FCNC:

valid at some energy scale $Q_0 \lesssim M_{\rm Planck}$

- scalar masses diagonal in the flavor space

$$\left(m^2_{\tilde{q}}\right)^2_{ij} = m^2_{Q_i}\delta_{ij} \quad \left(m^2_{\tilde{u}^c}\right)^2_{ij} = m^2_{U_i}\delta_{ij} \quad \left(m^2_{\tilde{d}^c}\right)^2_{ij} = m^2_{D_i}\delta_{ij}$$

$$\left(m^2_{\tilde{\ell}}\right)^2_{ij} = m^2_{L_i}\delta_{ij} \quad \left(m^2_{\tilde{e}^c}\right)^2_{ij} = m^2_{E_i}\delta_{ij}$$

- A-matrices proportional to Yukawa couplings

$$(\mathbf{A_u})_{ij} = A_U\,(\lambda_u)_{ij} \quad (\mathbf{A_d})_{ij} = A_D\,(\lambda_d)_{ij} \quad (\mathbf{A_e})_{ij} = A_L\,(\lambda_e)_{ij}$$

in addition soft Higgs scalar masses $m_1^2|H_1|^2 + m_2^2|H_2|^2 + (B\mu H_1 H_2 + \mathrm{h.c.})$

$+$ gaugino masses $M_3,\ M_2,\ M_1 \Rightarrow 24$ parameters

minimal sugra: $m_0, m_{1/2}, A, B \quad (m^2_{1,2} = \mu^2 + m_0^2)$

Electroweak (EW) symmetry breaking

$$\mathcal{V}_{\rm neutral} = m_1^2|H_1^0|^2 + m_2^2|H_2^0|^2 + B\mu(H_1^0H_2^0 + {\rm h.c.}) + \frac{g_2^2+g_Y^2}{8}\left(|H_1^0|^2 - |H_2^0|^2\right)^2$$

- stability along $|H_1^0| = |H_2^0| \Rightarrow m_1^2 + m_2^2 > 2B\mu$
- EW symmetry breaking $\Rightarrow m_1^2 m_2^2 - B^2\mu^2 < 0$

Radiative symmetry breaking:

start with all scalar masses positive and $m_1^2 m_2^2 - B^2\mu^2 > 0$ at high energies

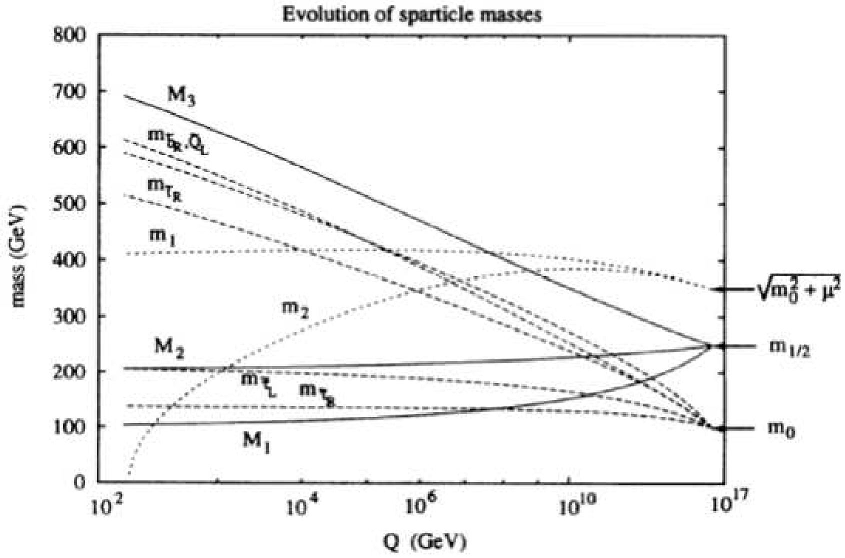
renormalization group evolution $\Rightarrow m_2^2$ is driven negative at low scale

$$\frac{dm_2^2}{d\ln Q} = \frac{3\lambda_t^2}{8\pi^2}\left(m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2 + |A_t|^2 + m_2^2\right) + \cdots$$

$$\frac{dm_{\tilde{t}_L}^2}{d\ln Q} = -\frac{16}{24\pi^2}g_3^2 M_3^2 + \frac{\lambda_t^2}{8\pi^2}\left(m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2 + |A_t|^2 + m_2^2 - \mu^2\right) + \cdots$$

- QCD effects: stop becomes heavier in the IR $\Rightarrow$ color unbroken
- top Yukawa: drives m_2^2 negative λ_t must be $\mathcal{O}(1)$

minimal SUGRA



Higgs mass

parameters: $m_1, m_2, B\mu \;\rightarrow\; \langle H_1^0 \rangle = v_1, \langle H_2^0 \rangle = v_2, m_A$

$m_Z, \tan\beta = v_2/v_1$

$m_Z^2 = \frac{g_2^2+g_Y^2}{2} v^2 \quad v = \sqrt{v_1^2 + v_2^2}$

$$m_A^2 = m_1^2 + m_2^2 \qquad m_A^2 \sin 2\beta = -2B\mu \qquad m_Z^2 = 2\frac{m_1^2 - m_2^2 \tan_\beta^2}{\tan^2\beta - 1}$$

$$m_{H^+}^2 = m_A^2 + m_W^2 \quad m_{H,h}^2 = \frac{1}{2}\left\{ m_A^2 + m_Z^2 \pm \sqrt{(m_A^2 - m_Z^2)^2 + 4m_A^2 m_Z^2 \sin^2 2\beta} \right\}$$

$$\Rightarrow \; m_h < m_A < m_H \qquad m_h < m_Z$$

However important quantum corrections from top/stop loop:

$\delta m_h^2 = \frac{3}{\pi}\frac{m_t^4}{m_W^2}\sin^2\!\beta \ln \frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}$ $\Rightarrow$ lightest Higgs bound: $m_h \lesssim 130$ GeV

sparticle spectrum

- sfermions:
  - first two generations: neglect Yukawa couplings $\Rightarrow$

    D-term contributions + soft masses $\Rightarrow$

    $m^2_{\tilde{e}} - m^2_{\tilde{\nu}} = |\cos 2\beta| m^2_Z$ $\rightarrow$ tan β determination
  - 3rd generation: 2×2 mass matrix for $\tilde{t}_L,\ \tilde{t}_R \rightarrow \tilde{t}_1,\ \tilde{t}_2$

    similarly $\tilde{b}_L,\ \tilde{b}_R \rightarrow \tilde{b}_1,\ \tilde{b}_2$

- wino-bino-higgsino mixing $\Rightarrow$ charginos + neutralinos
  - charginos $\tilde{W}^{\pm},\ \tilde{H}^{\pm}$: $\begin{pmatrix} M_2 & \sqrt{2} m_W \sin\beta \\ \sqrt{2} m_W \cos\beta & \mu \end{pmatrix}$

    $\rightarrow$ two Dirac states: $\tilde{C}_1^{\pm}, \tilde{C}_2^{\pm}$
  - neutralinos $\tilde{W}^3,\ \tilde{B},\ \tilde{H}_1^0,\ \tilde{H}_2^0$: 4×4 mixing matrix $\Rightarrow \tilde{N}_i \ \ i = 1, \dots, 4$