Introduction to Supersymmetry

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Lectures 1, 2

- Motivations, the problem of mass hierarchy, main BSM proposals
- Supersymmetry: formalism

  transformations, algebra, representations,

  superspace, chiral and vector super fields, invariant actions,

  R-symmetry, extended supersymmetry,

  spontaneous supersymmetry breaking

Standard Model of electroweak + strong forces

- Quantum Field Theory   Quantum Mechanics + Special Relativity
- Principle: gauge invariance $U(1) \times SU(2) \times SU(3)$

Very accurate description of physics at present energies   17 parameters

1. mediators of gauge interactions (vectors): photon, $W^{\pm}$, Z + **8 gluons**
2. matter (fermions): (leptons + quarks) × 3

   electron, positron, neutrino   (up, down) **3 colors**
3. Higgs sector: new scalar(s) particle(s):
   - break the EW symmetry $U(1) \times SU(2) \rightarrow U(1)_{\gamma}$ at $v = 246$ GeV
   - generate mass for all elementary particles

Standard Model

$$\mathcal{L}_{\mathrm{SM}} = -\frac{1}{2}\mathrm{tr}\underset{\nearrow}{F^2_{\mu\nu}} + \bar{\psi}\not{D}\psi \underset{\uparrow}{+} \bar{\psi}YH\psi - |DH|^2 \underset{\uparrow}{-} V(H)$$

Three sectors: Forces Matter Higgs

1. Forces: uniquely determined by geometry (given the gauge group)
2. Matter: fixed by the representations (discrete arbitrariness)
3. Scalar (Higgs) sector for EW symmetry breaking:

arbitrary (no guidance)

minimal (1 scalar) or more complex?

$$V(H) = -\mu^2|H|^2 + \lambda\left(|H|^2\right)^2 \quad [8]$$

μ: the only mass parameter creating the 'hierarchy' problem

July 4th 2012 The discovery of a new particle

International Herald Tribune

Discovery upends world of physics

The New York Times

Physicists Find Elusive Particle Seen as Key to Universe

The Economist

A giant leap for science

Finding the Higgs boson

ヒッグス粒子発見か

新素粒子検出 年内に結論

Milhares de moradores de bairros sociais em risco de perderem RSI

Le Monde

Science : la matière dévoilée

The Gazette

EL PAIS

Hallada la partícula clave para la comprensión del universo

MK

ПОСЛЕДНИЙ КИРПИЧ В СТЕНУ МИРОЗДАНИЯ

ALGEMEEN DAGBLAD

EINDELIJK GELIJK NA 48 JAAR

Frankfurter Allgemeine

CHINA DAILY

IMPORTANT MATTER Scientists claim to have discovered 'God particle'

THE TIMES OF INDIA

Big bang moment: Scientists may have found 'God particle'

THE HINDU

Elusive particle found, looks like Higgs boson

CORRIERE DELLA SERA

La particella che può svelare i segreti dell'universo

gazeta WYBORCZA.PL

Cząstkę Higgsa fizycy najpierw wymyślili, potem szukali 40 lat

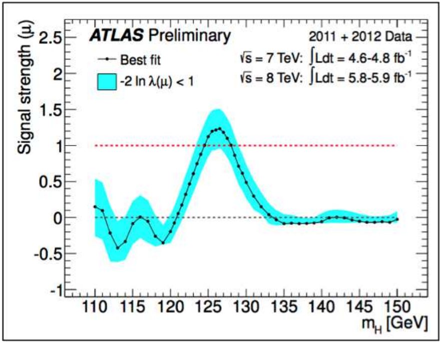
BOSKA MASA

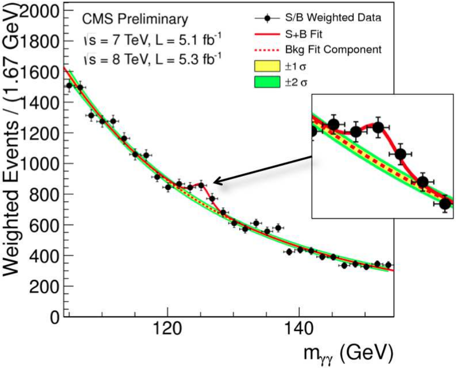
আনন্দবাজার পত্রিকা

বিজ্ঞানের 'ঈশ্বর' দর্শন

'পেয়েছি, যা খুঁজছিলাম'

Higgs boson discovery

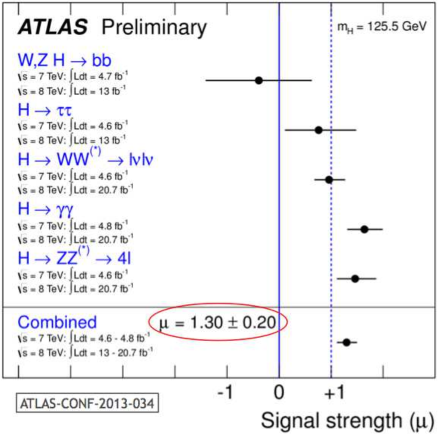


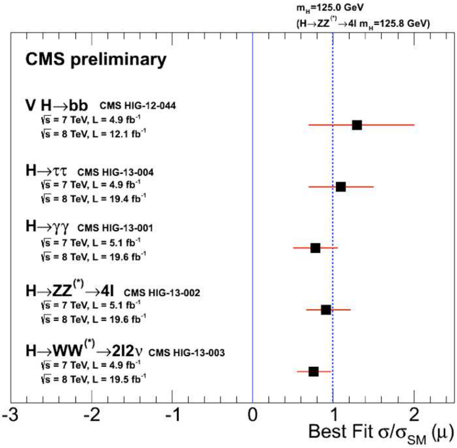


$m_H = 125.5 \pm 0.2\,(\mathrm{stat.}) \pm 0.5\,(\mathrm{syst.})$

$m_H = 125.7 \pm 0.3 \pm 0.3$ GeV

Couplings of the new boson vs SM





exclusion : spin 2 and pseudoscalar at $\gtrsim$ 95% CL

Agreement with Standard Model expectation at $\sim 2\,\sigma$

François Englert ↓ Peter Higgs ↓ Nobel Prize of Physics 2013

some remarks

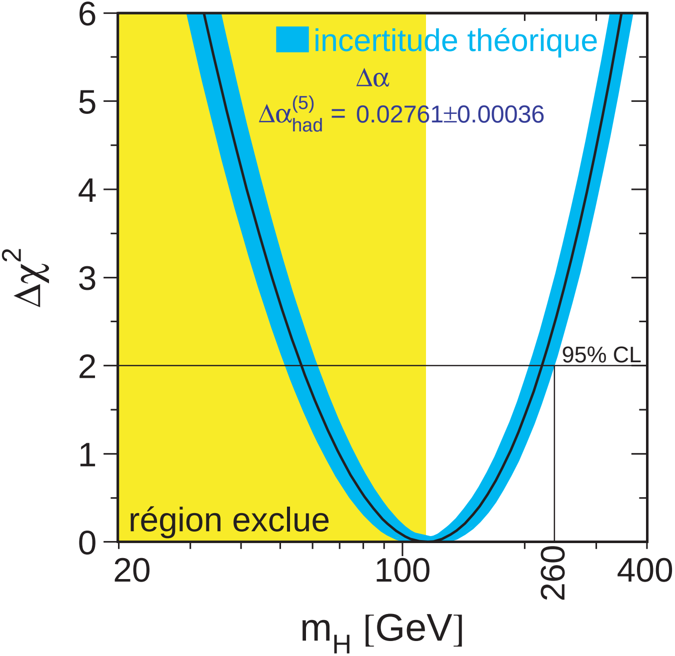
Englert-Brout-Higgs mechanism

Englert-Brout; Higgs; Guralnik-Hagen-Kibble '64

Its discovery was one of the main goals of LHC

Higgs scalar discovery around 125 GeV :

- consistent with expectation from precision tests of the SM
- favors perturbative physics quartic coupling $\lambda = m_H^2/v^2 \simeq 1/8$ [3]
- very important to measure Higgs couplings
- 1st elementary scalar in nature signaling perhaps more to come [10]


🟦 incertitude théorique
Δα
$\Delta\alpha^{(5)}_{had} = 0.02761 \pm 0.00036$
95% CL
région exclue
$\Delta\chi^2$
6
5
4
3
2
1
0
20
100
260
400
m_H [GeV]

Why Beyond the Standard Model?

Theory reasons:

- Include gravity
- Charge quantization
- Mass hierarchies: $m_{\mathrm{electron}}/m_{\mathrm{top}} \simeq 10^{-5}$ $M_W/M_{\mathrm{Planck}} \simeq 10^{-16}$

Experimental reasons:

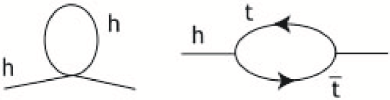
- Neutrino masses
  - Dirac type ⇒ new states
  - Majorana type ⇒ Lepton number violation, new states
- Unification of gauge couplings
- Dark matter [13]

Mass hierarchy problem

Higgs mass: very sensitive to high energy physics

1-loop radiative corrections:

dominant contributions:



$$\mu_{\text{eff}}^2 = \mu_{\text{bare}}^2 + \left(\frac{\lambda}{8\pi^2} - \frac{3\lambda_t^2}{8\pi^2} \right) \Lambda^2 + \cdots$$

UV cutoff: $\int^{\Lambda} \frac{d^4k}{k^2}$ scale of new physics

- sign of μ^2 can easily change
- High-energy validity of the Standard Model $\Rightarrow \Lambda >> \mathcal{O}(100)$ GeV

  $\Rightarrow$ "unatural" fine-tuning between μ_{bare}^2 and radiative corrections

  order by order

Mass hierarchy problem

example: $\Lambda \sim \mathcal{O}(M_{\rm Planck}) \sim 10^{19}$ GeV, loop factor $\sim 10^{-2}$

$$\Rightarrow \mu^2_{1\ \rm loop} \sim 10^{-2} \times 10^{38} = \pm 10^{36}\ (\text{GeV})^2$$

need $\mu^2_{\rm bare} \sim \mp 10^{36}\ (\text{GeV})^2 - 10^4\ (\text{GeV})^2$

- adjustment at the level of 1 part per 10^{32} $\mu^2_{\rm bare}/\mu^2_{1\ \rm loop} = -1 \mp 10^{-32}$
- new adjustment at the next order, etc

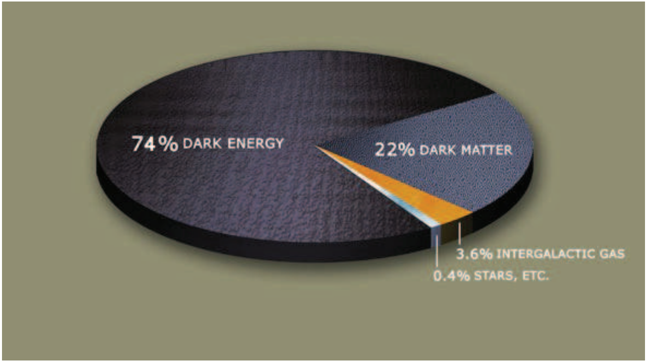
  highest order N: $\left(10^{-2}\right)^N \times 10^{38} \lesssim 10^4 \Rightarrow N \gtrsim 18$ loops !
- no fine tuning : $10^{-2}\Lambda^2 \lesssim 10^4\ (\text{GeV})^2 \Rightarrow \Lambda \lesssim 1$ TeV

  → new physics within LHC range ! [10]

What our Universe is made of ?

- Ordinary matter: only a tiny fraction
- Non-luminous (dark) matter: $\sim 25\%$

  Natural explanation: new stable Weakly Interacting Massive Particle in the LHC energy region
- Unknown relativistic dark energy: $\sim 70\%$



Directions Beyond the Standard Model

guidance the mass hierarchy

1. Compositeness
2. Symmetry: - supersymmetry
   - little Higgs*
   - conformal* [16]
   - higher dim gauge field*

   *need UV completion
3. Low UV cutoff: - low scale gravity* ⇛ · large extra dimensions
   · warped extra dimensions
   - low string scale ⇛ · low scale gravity
     · ultra weak string coupling
   - large N degrees of freedom
4. Live with the hierarchy: landscape of vacua, environmental selection
   → split supersymmetry [18]

Compositeness

strong dynamics at $\sim$ TeV $\Rrightarrow$ Higgs bound state of fermion bilinears

as the pions in QCD and chiral symmetry breaking

→ concrete proposal: technicolor

generic models $\Rrightarrow$ · FCNC

· conflict with EW precision data

$\Rrightarrow$ highly disfavored

126 GeV Higgs mass $\Rrightarrow$ perturbative interactions

Symmetry

Examples of naturally small masses

- Fermions: chiral symmetry

$$\mathcal{L}_F = i\bar{\psi}_L \not{D} \psi_L + i\bar{\psi}_R \not{D} \psi_R + m\left(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L\right)$$

$m = 0$ respects: $\psi_L \to \psi_L \quad ; \quad \psi_R \to e^{i\theta}\psi_R$

$\Rightarrow$ radiative corrections: $\delta m \propto g^2 m$ $\qquad g$: gauge coupling

e.g. QED: $\psi \equiv$ electron, $g \equiv e$

no $g^2\Lambda$ term: linear divergence cancels between electron and positron

in a relativistic quantum field theory

- Vector bosons: gauge symmetry

$$\mathcal{L}_V = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2}m^2 A_\mu^2$$

$m = 0$ respects: $A_\mu \to A_\mu + \partial_\mu \omega$ $\quad$ e.g. QED: photon mass vanishes

Symmetry

- Scalars: ?

\- Shift symmetry : Goldstone boson

$$\phi \to \phi + c \quad \Rightarrow \quad \mathcal{L}_{\rm GB}\left(\partial_\mu \phi\right) = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{\Lambda^4}\left[(\partial_\mu \phi)^2\right]^2 + \cdots$$

$\Rightarrow$ Higgs quartic coupling should vanish to lowest order

Higgs : pseudo-Goldstone boson? → Little Higgs models

symmetry broken by new gauge interactions

\- scale invariance: $x^\mu \to a x^\mu \quad \varphi_d \to a^{-d}\varphi(ax)$

conformal dimension

scalar field: $d = 1$

$$\mathcal{L}_{\rm S} = \frac{1}{2}(\partial_\mu \phi)^2 + \lambda \phi^4 \quad \text{invariant}$$

however broken hardly by radiative corrections renormalization scale

→ embed SM in a conformal invariant theory ? [14]

Mass hierarchy problem: protect scalar masses from quadratic divergences

A possible strategy:

1) connect scalars to fermions or gauge fields postulating new symmetries

2) use chiral or gauge symmetry to protect their mass

- $\delta\phi = \xi\psi \quad \Rightarrow$ supersymmetry
- $\delta\phi = \epsilon A \quad \Rightarrow$ extra dimensions

component of a higher-dimensional gauge field

2-component Weyl spinors

spin-1/2 irreps of Lorentz group: 2-dim Left and Right

$$\chi_{L,R} \to \left(1 - i\vec{\alpha}\cdot\vec{\sigma} \mp \vec{\beta}\cdot\vec{\sigma}\right)\chi_{L,R} \;\Rightarrow\; L : \chi_\alpha \,,\; R : \chi_{\dot{\alpha}}$$

infinitesimal rotation ↗ boost ↗ ↖ Pauli matrices

charge conjugation matrix $C = i\sigma^2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

$C : L \to R$ $C\chi^* \equiv \bar{\chi}^c$ transforms as Right parity: $\chi \to \bar{\chi}^c$ $\Rightarrow$

describe R-spinor as charge conjugate of a L-spinor

Dirac-Weyl basis: $\psi = \begin{pmatrix} \chi_\alpha \\ \bar{\eta}^c_{\dot{\alpha}} \end{pmatrix}$ $\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}$ $\gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

$\sigma^\mu_{\alpha\dot{\alpha}} = (1, \vec{\sigma})$ $\bar{\sigma}^\mu_{\dot{\alpha}\alpha} = (1, -\vec{\sigma}^*)$ $C\sigma^\mu = (\bar{\sigma}^\mu)^T C$

Dirac and Majorana masses

$\psi_L = \begin{pmatrix} \chi_\alpha \\ 0 \end{pmatrix} \quad \psi_R = \begin{pmatrix} 0 \\ \bar{\eta}^c_{\dot{\alpha}} \end{pmatrix} \quad \Rightarrow$

$$\begin{aligned} \mathcal{L}_F &= i\bar{\psi}\partial\!\!\!/\psi - m\bar{\psi}\psi \\ &= i\chi^*\,\bar{\sigma}^\mu\partial_\mu\chi + i\eta^*\,\bar{\sigma}^\mu\partial_\mu\eta - (m\chi\eta + \text{h.c.}) \quad \chi\eta \equiv \chi^T C\eta = \epsilon_{\alpha\beta}\chi^\alpha\eta\beta \end{aligned}$$

charge or fermion number $\chi : +1 \quad \eta : -1 \quad \Rightarrow$

- Dirac mass invariant
- Majorana mass: $\eta = \chi \;\Rightarrow\; \epsilon_{\alpha\beta}\chi^\alpha\chi\beta$ violates charge

  Majorana fermion: $\psi = \begin{pmatrix} \chi_\alpha \\ \bar{\chi}^c_{\dot{\alpha}} \end{pmatrix}$

Supersymmetry

$\delta\phi = \xi\psi$ ξ: Weyl spinor $\Rightarrow$ conserved spinorial charge Q_α

$$[Q_\alpha, \phi] = \psi \quad , \quad [Q_\alpha, H] = 0 \quad \Rightarrow \quad \left\{Q_\alpha, \bar{Q}_{\dot\alpha}\right\} = -2\sigma^\mu_{\alpha\dot\alpha} R_\mu$$

anticommutator ↗

if R_μ conserved charge besides $T_{\mu\nu}$ (P^μ + angular momentum)

$\Rightarrow$ trivial theory with no scattering Coleman-Mandula

e.g. 4-point scattering for fixed center of mass s

depends only on the scattering angle θ

if extra charge $R^\mu \Rightarrow$ trivial

$\Rightarrow R_\mu = P_\mu$: supersymmetry = "$\sqrt{\text{translations}}$"

SUSY transformations

$$
\begin{aligned}
\delta\phi &= \sqrt{2}\xi\psi \\
\delta\psi &= -i\sqrt{2}\left(\sigma^\mu\bar{\xi}\right)_\alpha \partial_\mu\phi - \sqrt{2}F\xi \\
\delta F &= -i\sqrt{2}\left(\partial_\mu\psi\right)\sigma^\mu\bar{\xi}
\end{aligned}
$$

F: complex auxiliary field to close the SUSY algebra off-shell

- ϕ, F: complex
- chirality to scalars: $\phi \leftrightarrow$ left $\quad \phi^* \leftrightarrow$ right

Exercise: $\delta_1\delta_2 - \delta_2\delta_1 = -2i\left(\xi_1\sigma^\mu\bar{\xi}_2 - \xi_2\sigma^\mu\bar{\xi}_1\right)\partial_\mu$ on all fields

$\nearrow$

$$\left\{Q_\alpha, \bar{Q}_{\dot\alpha}\right\} = -2\sigma^\mu_{\alpha\dot\alpha}P_\mu$$

SUSY algebra

SUSY algebra: ‘Graded’ Poincaré:

$$\left\{Q_\alpha, \bar{Q}_{\dot\alpha}\right\} = -2\sigma^\mu_{\alpha\dot\alpha} P_\mu \qquad \{Q, Q\} = \{\bar{Q}, \bar{Q}\} = [P_\mu, Q] = 0$$

$$[Q_\alpha, M^{\mu\nu}] = i\sigma^{\mu\nu}_{\alpha\beta} Q_\beta$$

$M^{\mu\nu}$: Lorentz transformations

$$\nwarrow$$

$$[\sigma^\mu, \sigma^\nu]$$

generalization for many supercharges $\Rightarrow$ N extended SUSY

$$\left\{Q^i_\alpha, \bar{Q}^j_{\dot\alpha}\right\} = -2\sigma^\mu_{\alpha\dot\alpha} P_\mu \delta^{ij} \qquad \left\{Q^i_\alpha, Q^j_\beta\right\} = \epsilon_{\alpha\beta} Z^{ij} \qquad i, j = 1, \dots N$$

Z^{ij}: antisymmetric central charge $[Z, Q] = 0$

SUSY representations

Q_α has spin 1/2 $\quad\Rightarrow\quad Q|J\rangle = |J \pm 1/2\rangle$ $\qquad$ J: spin

- same mass: $[Q, P_\mu] = 0 \quad\Rightarrow\quad M_J = M_{J\pm 1/2}$
- same number of fermionic and bosonic degrees of freedom

supermultiplets characterized by their higher spin → massless:

- chiral: $\begin{pmatrix} \chi_\alpha \\ \phi \end{pmatrix}$ $\begin{matrix} \text{Weyl spinor} \\ \text{complex scalar} \end{matrix}$
- vector: $\begin{pmatrix} V_\mu \\ \lambda_\alpha \end{pmatrix}$ $\begin{matrix} \text{spin}\,1 \\ \text{Majorana spinor} \end{matrix}$
- gravity: $\begin{pmatrix} g_{\mu\nu} \\ \psi^\mu_\alpha \end{pmatrix}$ $\begin{matrix} \text{spin}\,2 \\ \text{Majorana spin}\,3/2 \end{matrix}$
- spin 3/2: $\begin{pmatrix} \psi^\mu_\alpha \\ V_\mu \end{pmatrix}$ $\begin{matrix} \text{spin}\,3/2 \\ \text{spin}\,1 \end{matrix}$ $\qquad$ extended supergravites

Massive supermultiplets

- spin-1/2: - chiral with Majorana mass

  - 2 chiral with Dirac mass
- spin-1: 1 vector + 1 chiral

  spin-1 + Dirac spinor + real scalar

Superspace

$\left(P^{\mu}, Q, \bar{Q}\right) \quad \longrightarrow \quad \left(x^{\mu}, \theta_{\alpha}, \bar{\theta}_{\dot{\alpha}}\right)$ $\theta, \bar{\theta}$: Grassmann variables

Grassmann algebra: $x_1, x_2, \ldots, x_n \;\rightarrow\; \theta_1, \theta_2, \ldots, \theta_n$ 1-component

anticommuting: $[x_i, x_j] = 0$ $\{\theta_i, \theta_j\} = 0 \quad \Rightarrow$

$\theta_i^2 = 0 \;\Rightarrow\; f(\theta) = f_0 + f_1\theta$

$$f(\theta_1, \ldots, \theta_n) = f_0 + f_i\theta_i + f_{ij}\,\theta_i\theta_j + \cdots + f_{1\cdots n}\,\theta_1\theta_2\cdots\theta_n$$

integration: analog of $\int_{-\infty}^{\infty} dx$

translation invariance: $\int d\theta f(\theta) = \int d\theta f(\theta + \varepsilon) \;\Rightarrow$

$\int d\theta = 0 \;\; \int d\theta\theta = 1 \;\; \int d\theta f(\theta) = f_1$ integration $\int d\theta \equiv$ derivation $\frac{d}{d\theta}$

$\int d\theta_1 \cdots d\theta_n f(\theta_1, \cdots, \theta_n) = f_{1\cdots n}$

super-covariant derivatives

SUSY transformation $\equiv$ translation in superspace

SUSY group element: $G(x,\theta,\bar\theta) = e^{i(\theta Q + \bar\theta\bar Q + x^\mu P_\mu)}$

multiplication using $e^A e^B = e^{A+B+\frac{1}{2}[A,B]}$:

$$G(x,\theta,\bar\theta)G(y,\eta,\bar\eta) = G(x^\mu + y^\mu + i\theta\underset{\nearrow \frac{1}{2}\theta\{Q,\bar Q\}\bar\eta}{\sigma^\mu}\bar\eta - i\eta\sigma^\mu\bar\theta, \theta+\eta, \bar\theta+\bar\eta) \;\Rightarrow$$

$$\begin{aligned} Q_\alpha &= \;\; i\left[\frac{\partial}{\partial\theta^\alpha} - i\left(\sigma^\mu\bar\theta\right)_\alpha \partial_\mu\right] & P_\mu = i\partial_\mu \quad & \left\{Q_\alpha, \bar Q_{\dot\alpha}\right\} = -2i\sigma^\mu_{\alpha\dot\alpha}\partial_\mu \\ \bar Q_{\dot\alpha} &= -i\left[\frac{\partial}{\partial\bar\theta^{\dot\alpha}} - i\left(\theta\sigma^\mu\right)_{\dot\alpha}\partial_\mu\right] & & \{Q_\alpha, Q_\beta\} = 0 \end{aligned}$$

super-covariant derivatives

commute with SUSY transformations:

$$\{D, Q\} = \{D, \bar{Q}\} = \{\bar{D}, Q\} = \{\bar{D}, \bar{Q}\} = 0 \Rightarrow$$

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\left(\sigma^\mu\bar{\theta}\right)_\alpha \partial_\mu \qquad \left\{D_\alpha, \bar{D}_{\dot\alpha}\right\} = 2i\sigma^\mu_{\alpha\dot\alpha}\partial_\mu$$

$$\bar{D}_{\dot\alpha} = \frac{\partial}{\partial\bar{\theta}^{\dot\alpha}} + i\left(\theta\sigma^\mu\right)_{\dot\alpha} \partial_\mu \qquad \{D, D\} = D^3 = \bar{D}^3 = 0$$

Chiral superfield

$\Phi(x,\theta,\bar\theta)$ satisfying: $\bar D_{\dot\alpha}\Phi = 0 \;\Rightarrow\; \Phi(y,\theta) \qquad y = x + i\theta\sigma\bar\theta$

$\Rightarrow$ chiral superspace $\{y^\mu, \theta_\alpha\}$

similar antichiral superfield $\Phi^\dagger(\bar y, \bar\theta) \qquad \bar y = x - i\theta\sigma\bar\theta$

expansion in components:

$$\Phi(y,\theta) = \phi(y) + \sqrt{2}\theta\psi(y) - \theta^2 F(y) \qquad \theta^2 = \frac{1}{2}\theta_\alpha\theta_\beta\epsilon^{\alpha\beta}$$

mass dimension $[\theta] = -1/2$

Exercise: From a translation in superspace derive

the SUSY transformations for the components (ϕ, ψ, F)

SUSY action of chiral multiplets

$$\mathcal{L}_K = \int d^2\theta d^2\bar{\theta}\, \Phi^\dagger \Phi = |\partial\phi|^2 + i\bar{\psi}\bar{\sigma}^\mu \partial_\mu \psi + |F|^2$$

- free complex scalar + Weyl fermion
- $F = 0$ by equations of motion

$$\mathcal{L}_W = \int d^2\theta W(\Phi) = F\frac{\partial W}{\partial\phi} - \psi\psi\frac{\partial^2 W}{(\partial\phi)^2} \qquad \psi\psi = \psi_\alpha\psi_\beta\epsilon^{\alpha\beta}$$

superpotential W : arbitrary analytic function

$\mathcal{L} = \mathcal{L}_K + \mathcal{L}_W + \mathcal{L}_{W^\dagger} \quad \Rightarrow \quad F^* = -\frac{\partial W}{\partial\phi} \Rightarrow$

- scalar potential $\mathcal{V} = \left|\frac{\partial W}{\partial\phi}\right|^2$
- Yukawa interaction $\frac{\partial^2 W}{(\partial\phi)^2}\psi\psi + \mathrm{c.c.}$

renormalizability $\Rightarrow$ W at most cubic

$$W(\Phi_i) = \frac{1}{2}M_{ij}\Phi_i\Phi_j + \frac{1}{3}\lambda_{ijk}\Phi_i\Phi_j\Phi_k \quad \Rightarrow$$

- $V_{\rm scalar} = \sum_i |M_{ij}\phi_j + \lambda_{ijk}\phi_j\phi_k|^2$
- $\mathcal{L}_{\rm Yukawa} = -M_{ij}\psi_i\psi_j - \lambda_{ijk}\phi_i\psi_j\psi_k$

generalization: $\mathcal{L} = \int d^4\theta K\left(\Phi, \Phi^\dagger\right) + \left\{\int d^2\theta W(\Phi) + \text{c.c.}\right\}$

Kähler potential K : arbitrary real function

$$\Rightarrow \quad \mathcal{L}_K = g_{i\bar{j}}(\partial\phi^i)(\partial\bar{\phi}^{\bar{j}}) + \cdots$$

Kähler metric $g_{i\bar{j}} = \frac{\partial}{\partial\phi^i}\frac{\partial}{\partial\bar{\phi}^{\bar{j}}}K(\phi,\bar{\phi})$ → Kähler manifold

Vector superfield

$V(x,\theta,\bar\theta)$: real

abelian gauge transformation: $\delta V = \Lambda + \Lambda^\dagger$ Λ: chiral

super-gauge fixing $\Rightarrow$ Wess-Zumino gauge:

$$V = \theta\sigma^\mu\bar\theta A_\mu + i\bar\theta^2\theta\lambda - i\theta^2\bar\theta\bar\lambda + \tfrac{1}{2}\bar\theta^2\theta^2\mathrm{D}$$

$\nearrow$ gauge field $\nwarrow$ gaugino $\nwarrow$ real auxiliary field

$$\delta A^\mu = \bar\xi\bar\sigma^\mu\lambda + \bar\lambda\bar\sigma^\mu\xi \qquad \delta\lambda = \left(i\sigma^{\mu\nu}F_{\mu\nu} + \mathrm{D}\right)\xi$$

$$\delta\mathrm{D} = -i\bar\xi\bar\sigma^\mu\partial_\mu\lambda + i(\partial_\mu\bar\lambda)\bar\sigma^\mu\xi$$

chiral field strength: $\mathcal{W}_\alpha = -\frac{1}{4}\bar D^2 D_\alpha V$ $\bar D\mathcal{W}_\alpha = 0$

$$\mathcal{W}_\alpha = -i\lambda_\alpha + \theta_\alpha\mathrm{D} + \frac{i}{4}\left(\theta\sigma^{\mu\nu}\right)_\alpha F_{\mu\nu} + \theta^2\left(\sigma^\mu\partial_\mu\bar\lambda\right)_\alpha$$

SUSY gauge actions

- Kinetic terms chiral: $\mathcal{L}_{\rm gauge} = \frac{1}{4}\int d^2\theta\, \mathcal{W}^2 + {\rm h.c.}$

$$= -\frac{1}{4}F_{\mu\nu}^2 + \frac{i}{2}\lambda\sigma^\mu \overset{\leftrightarrow}{\partial}_\mu \bar{\lambda} + \frac{1}{2}{\rm D}^2$$

- Covariant derivatives in matter action

gauge transformation $\delta\Phi_q = q\,\Lambda\Phi_q$ $\quad q$: charge $\quad \Rightarrow$

$$\Phi_q^\dagger\Phi_q \quad \longrightarrow \quad \Phi_q^\dagger e^{-qV}\Phi_q$$

additional contribution to the scalar potential: $\frac{1}{2g^2}{\rm D}^2 - q\,|\phi_q|^2\,{\rm D} \quad \Rightarrow$

$${\rm D} = g^2\sum_i q_i\,|\phi_i|^2 \qquad \mathcal{V}_{\rm gauge} = \frac{g^2}{2}\left(\sum_i q_i\,|\phi_i|^2\right)^2$$

- Non abelian case: $V \to V_a\, t^a \leftarrow$ gauge group generators

$$\Rightarrow \quad {\rm Tr}\,\mathcal{W}^2 \qquad q_i\,|\phi_i|^2 \to \bar{\phi}_i t^a \phi_i$$

- generalization: $\mathcal{L}_{\rm gauge} = \frac{1}{4}\int d^2\theta f(\Phi)\,\mathcal{W}^2 + {\rm h.c.}$ $\quad f$: analytic

R-symmetry

Chiral rotation of superspace coordinates: $\theta \to e^{i\omega}\theta$ R-charge $= 1$

$\bar\theta \to e^{-i\omega}\bar\theta$ R-charge $= -1$

$\Rightarrow$ superfield components: $\Delta Q_R \,|_{\mathrm{bosons-fermions}} = \pm 1$

e.g. R-charges: $\Phi_q = \phi_q + \sqrt{2}\theta_1\psi_{q-1} - \theta_1^2 F_{q-2}$

Chiral measure $d^2\theta$ has charge -2 $\int d^2\theta\,\theta^2 = 1$ $\Rightarrow$

- gauge superfield $\mathcal{W}$: R-charge $+1 =$ for gauginos
- superpotential W: charge $+2 \Rightarrow$ constraints on charges of Φ's

Renormalization properties

- Non renormalization of the superpotential

- Wave function renormalization of matter fields

  heuristic proof: promote Yukawa coupling λ to background

  chiral superfield of R-charge $+2$

  $\Rightarrow$ loop corrections analytic in λ → violate R-charge

  in wave functions posible because $\lambda\lambda^\dagger$ dependence allowed

- gauge kinetic terms only one loop

  promote $1/g^2$ to a chiral superfield S and use

  perturbative invariance under imaginary shift $S \rightarrow S + ic$

- Non perturbative corrections break R-symmetry to a discrete group

multiplet of currents

supersymmetry invariance $\Rightarrow$ conserved supercurrent $S_{\mu\alpha}$

$$\delta_s \mathcal{L} = \partial^\mu \left(K_\mu \xi\right) \Rightarrow S_\mu = S_\mu^{(N)} - K_\mu$$

↖

$$\text{Noether current} = \sum_\phi \frac{\delta \mathcal{L}}{\delta \partial^\mu \phi} \delta_s \phi$$

exercise: Show that $S_{\mu\alpha} = \sqrt{2}(\partial_\nu \bar\phi)(\sigma^\nu \bar\sigma_\mu \psi)_\alpha + i\sqrt{2}\,\overline{\partial_\phi W}(\sigma_\mu \bar\psi^c)_\alpha$

$\delta_s S_\mu \sim (\sigma^\nu \bar\xi)\, T_{\mu\nu} + \dots \Rightarrow$ multiplet of currents $(S_\mu, T_{\mu\nu}, \dots)$

$$\delta_s S_\mu = i\bar\xi^{\dot\alpha} \left\{\bar Q_{\dot\alpha}, S_\mu\right\} \Rightarrow$$

$$\int d^3x\, \delta_s S_{0\alpha} = i\bar\xi^{\dot\alpha} \left\{\bar Q_{\dot\alpha}, Q_\alpha\right\} = -2i(\sigma^\nu \bar\xi)_\alpha P_\nu = \int d^3x \left[-2i(\sigma^\nu \bar\xi)_\alpha\right] T_{0\nu}$$

Similarly: R-current j_μ^R ↔ trace T_μ^μ

The Ferrara-Zumino supermultiplet

Real superfield $\mathcal{J}_{\alpha\dot{\alpha}} = (j^R_\mu, S_{\mu\beta}, T_{\mu\nu})$

$$\bar{D}^{\dot{\alpha}} \mathcal{J}_{\alpha\dot{\alpha}} = D_\alpha X \quad \bar{D}_{\dot{\alpha}} X = 0$$

$X = 0 \leftrightarrow$ superconformal invariance: $\partial^\mu j^R_\mu = \gamma^\mu S_\mu = T^\mu_\mu = 0$

8 bosonic + 8 fermionic components

$$j^R_\mu : 4 - 1 = 3\,,\ T_{\mu\nu} : 10 - 4 - 1 = 5\,,\ S_\mu : 16 - 4 - 4 = 8$$

$X \neq 0$: 12 bosonic + 12 fermionic components

$$\text{bosonic} \quad j^R_\mu : 4\,,\ T_{\mu\nu} : 10 - 4 = 6\,,\ X : 2$$

example: Wess-Zumino model

$$\mathcal{J}_{\alpha\dot{\alpha}} = 2g_{i\bar{j}}(D_\alpha \Phi^i)(\bar{D}_{\dot{\alpha}} \bar{\Phi}^{\bar{j}}) - \tfrac{2}{3}\left[D_\alpha, \bar{D}_{\dot{\alpha}}\right] K \quad , \quad X = 4W - \tfrac{1}{3}\bar{D}^2 K$$

Extended supersymetry

N supercharges $Q^i \quad i = 1, \ldots N$

- multiplets with spin$\leq 1 \Rightarrow N \leq 4 \leftarrow$ maximal global SUSY
- multiplets with spin$\leq 2 \Rightarrow N \leq 8 \leftarrow$ maximal supergravity
- dimension of massless reps $= (2) \times 2^N$
- dimension of massive reps $= (2) \times 2^{N+1}$

  in the presence of central charge $Z \neq 0 \Rightarrow$ short reps $\equiv$ massless

$N = 2$ massless multiplets $\equiv N = 1$ massive

- vector multiplet $=$ vector $+$ chiral of $N = 1$

  1 vector $+$ 1 Dirac spinor $+$ 1 complex scalar

- hypermultiplet $=$ 2 chiral of $N = 1$

  2 complex scalars $+$ 1 Dirac spinor

$N = 4$ massless multiplet: vector $=$ vector $+$ hyper of $N = 2$

1 vector $+$ 4 two-component spinors $+$ 6 real scalars

superfield off-shell formalism only for $N = 2$ vector multiplets

chiral $N = 2$ superspace $\equiv$ double of $N = 1$ superspace: $\theta, \tilde{\theta}$

gauge chiral multiplet$\big|_{N=2}\, \mathcal{A} = (\text{vector } \mathcal{W} + \text{chiral } A)_{N=1}$

$\mathcal{A}(y, \theta, \tilde{\theta}) = A(y, \theta) + i\sqrt{2}\tilde{\theta}\mathcal{W}(y, \theta) - \frac{1}{4}\tilde{\theta}^2\overline{DDA}(y, \theta)$

$$\mathcal{L}^{N=2}_{Maxwell} = -\frac{1}{8}\int d^2\theta\, d^2\tilde{\theta}\, \mathcal{A}^2 + h.c. = \int d^2\theta \left(\frac{1}{2}\mathcal{W}^2 - \frac{1}{4}A\overline{DDA}\right) + h.c.$$

generalization: analytic prepotential $f(\mathcal{A})$

$$-\frac{1}{8}\int d^2\theta\, d^2\tilde{\theta}\, f(\mathcal{A}) + h.c. = \frac{1}{4}\int d^2\theta \left[f''(A)\mathcal{W}^2 - \frac{1}{2}f'(A)\overline{DDA}\right] + h.c.$$

Supersymmetry breaking

Spontaneous SUSY breaking

SUSY algebra in vacuum ($\vec{P} = 0, P^0 = H$): $\langle 0|\{Q, \bar{Q}\}|0\rangle = 2\langle 0|H|0\rangle$

exact SUSY ⇔ zero energy ; broken SUSY ⇔ positive energy

Indeed: $\mathcal{V}_{\text{scalar}} = \sum_{\substack{\text{chiral}\\ \text{superfields}}} |F|^2 + \frac{1}{2} \sum_{\substack{\text{vector}\\ \text{superfields}}} D^2$

$\Rightarrow$ broken SUSY: $\langle F\rangle$ or $\langle D\rangle \neq 0$

- SUSY can be broken only by a VEV of an auxiliary field

  scalar VEVs alone do not break SUSY if $\langle F\rangle = \langle D\rangle = 0$

  $\delta\psi = -\sqrt{2}\,\langle F\rangle\,\xi + \cdots \qquad \delta\lambda = \langle D\rangle\,\xi + \cdots$

F-breaking: O'Raifeartaigh model

$$W = -g\phi_0\phi_1^2 + m\phi_1\phi_2 + M^2\phi_0$$

$$F_0^* = \frac{\partial W}{\partial\phi_0} = -g\phi_1^2 + M^2 = 0 \quad \leftarrow \text{SUSY condition}$$

$$F_1^* = -2g\phi_0\phi_1 + m\phi_2 = 0 \qquad F_2^* = m\phi_1 = 0$$

$F_0 = F_2 = 0$: impossible $\Rightarrow$ supersymmetry breaking

Minimization of the scalar potential $\mathcal{V}$:

- $\phi_1 = \phi_2 = 0$, ϕ_0 arbitrary

  $F_0 = M^2,\ F_1 = F_2 = 0$ $\qquad$ $\mathcal{V} = M^4$ $\qquad$ $\frac{m^2}{2g} > M^2$

- $|\phi_1|^2 = \frac{M^2}{g} - \frac{m^2}{2g^2}$, $\phi_2 = \frac{2g}{m}\phi_0\phi_1$, ϕ_0 arbitrary

  $F_0 = \frac{m^2}{2g},\ F_2 = m\phi_1,\ F_1 = 0$ $\qquad$ $\mathcal{V} = \frac{m^2}{g}M^2 - \frac{m^4}{4g^2}$ $\qquad$ $\frac{m^2}{2g} < M^2$

D-breaking: Fayet-Iliopoulos model

$\xi \int d^4\theta V$ is SUSY and gauge invariant if V abelian $\Rrightarrow$

$$\xi D \quad \to \quad D = \xi + e \sum_i q_i \left|\phi_i\right|^2 \qquad [\xi] = (\mathrm{mass})^2$$

Example: two massive chiral multiplets $\Phi_\pm$ with charges ± 1 under a $U(1)$

$$D = \xi + e\left(|\phi_+|^2 - |\phi_-|^2\right) = 0 \quad \leftarrow \text{SUSY condition}$$

$$W = m\phi_+\phi_- \quad \Rrightarrow \quad F_+^* = m\phi_- = 0 \qquad F_-^* = m\phi_+ = 0$$

SUSY conditions incompatible $\Rrightarrow$ supersymmetry breaking

Minimization of the scalar potential $\mathcal{V}$:

- $\phi_+ = 0,\ |\phi_-|^2 = \frac{e\xi - m^2}{e^2}$ $\quad F_- = 0,\ D = \frac{m^2}{e},\ F_+ = m\phi_-$

  $\mathcal{V} = -\frac{m^4}{2e^2} + \frac{\xi m^2}{e} \qquad \xi > \frac{m^2}{e}$

- $\phi_+ = \phi_- = 0 \quad F_+ = F_- = 0,\ D = \xi \quad \mathcal{V} = \frac{1}{2}\xi^2$ $\qquad$ otherwise